

# **$^3\text{He}$ Critical Point Measurements**

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## **Abstract**

A theoretical analysis of the effect on the critical exponents of measurements along near-critical isochore and isotherm paths was performed. The uncertainty in the specific heat exponent,  $\alpha$  and compressibility exponents,  $\gamma$  and  $\delta$  along these near critical paths was determined. The construction of a specific heat cryostat was completed during the last year. At this time, we are in the middle of a long duration low-temperature run to evaluate the capability of this experimental system. Results of preliminary studies of the drift and pulse techniques for measuring the specific heat at constant volume are presented. The required temperature resolution for the microgravity experiment was demonstrated.

## **Introduction**

The acronym MISTE (Microgravity Scaling Theory Experiment) has been given to the flight definition experiment to test the scaling hypothesis near the liquid-gas critical point of  $^3\text{He}$  in a microgravity environment. This proposed experiment [1] will determine the critical exponents  $\alpha$ ,  $\gamma$ , and  $\delta$  along the critical isochore and critical isotherm by performing a set of precision measurements of the specific heat at constant volume,  $C_V$ , and the isothermal compressibility,  $\kappa_T$ . A stringent test of the theoretical predictions for the measured critical exponents as well as the Renormalization Group scaling hypothesis between exponents will be performed.

## **Accuracy in Determining Critical Exponents**

The accuracy in determining critical exponents strongly depends on the precise experimental path used to perform the measurements. We have analyzed the effect of a near critical isochore or isotherm path on the measurement of the critical exponents  $\alpha$ ,  $\gamma$  and  $\delta$  in a zero gravity environment. The desired precision in the specific heat,  $C_V$ , and compressibility,  $\kappa_T$ , along a near critical isochore path is 1% down to a reduced temperature of  $t = 10^{-7}$ . A similar 1% precision is the goal for the compressibility along the critical isotherm to a reduced density of  $5 \times 10^{-4}$ . Calculations were performed using the Ginsburg-Landau-Wilson Hamiltonian for the liquid-gas universality class  $O(1)$ . The critical exponents along critical paths used in the calculation are those obtained from High Temperature Expansions [2]. The predictions presented here were also shown to be consistent with calculations using the restricted cubic model [3].

Figure 1a shows the expected asymptotic deviation in the specific heat at constant volume,  $C_V$ , for measurements at varying reduced temperature,  $t = T/T_c - 1$ , along the near critical densities  $\Delta\rho = \rho/\rho_c - 1 = 5 \times 10^{-4}$  and  $1 \times 10^{-3}$ . The deviation of  $C_V$  from critical isochore behavior is shown in Fig. 1b. One can obtain an effective critical exponent from a fit of the theoretically generated “data” to the expected critical asymptotic power law expression. Using this fitting procedure on the “data” in Fig. 1a, the critical exponent  $\alpha$  is predicted to shift by -0.31 and -1.2% for isochores that are 0.05 and 0.1% away from the critical isochore respectively.

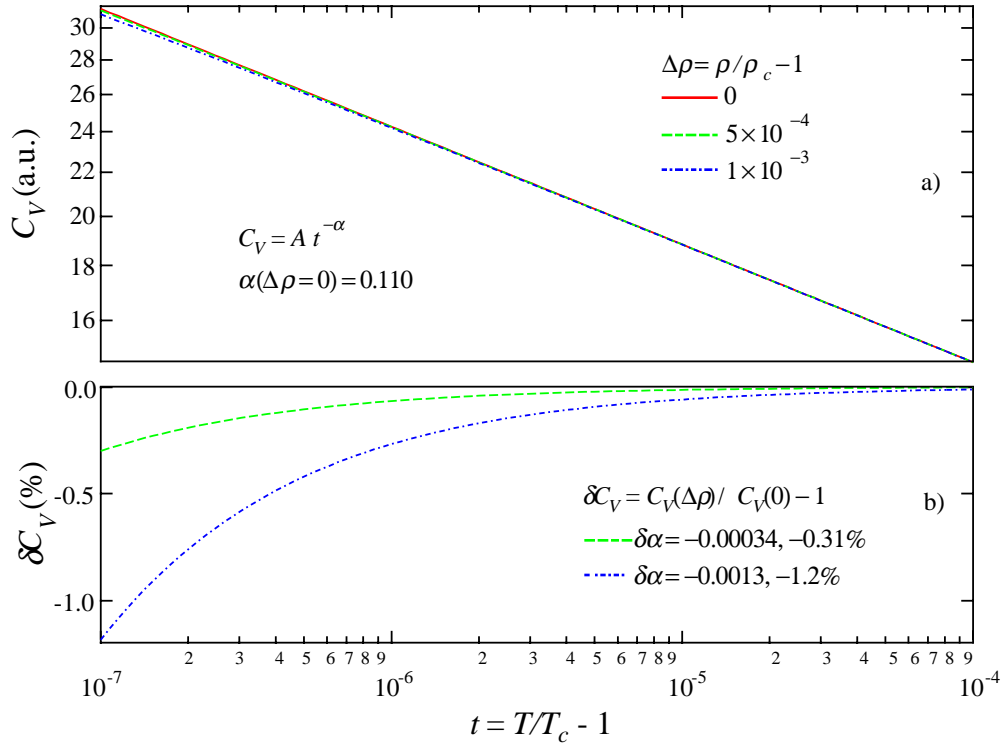


Fig. 1. Calculation of effective critical exponent  $\alpha$  for  $C_V$  along near critical isochore paths.

Similar calculations for the isothermal compressibility,  $\kappa_T$ , along near critical isochores are shown in Fig. 2. The deviation of the critical exponent  $\gamma$  from critical isochore behavior for reduced densities of 0.03 and 0.05% is predicted to be -0.11 and -0.31% respectively. These results for  $C_V$  and  $\kappa_T$  show that in order to obtain an experimental accuracy of 1% in the critical exponents  $\alpha$  and  $\gamma$ , deviations from the critical isochore must be less than 0.1%.

The calculations for the linear susceptibility,  $\chi_T$ , which is related to the isothermal compressibility,  $\kappa_T$ , by the expression  $\chi_T = \rho^2 \kappa_T$ , are shown along near critical isotherms in Fig. 3. For this case, because the critical exponent  $\delta$  is so large,  $\delta = 4.8$ , significant errors in the exponent can occur even for very small deviations from the critical isotherm. The effective change in the critical exponent  $\delta$  from critical isotherm behavior for reduced temperatures  $t = 5 \times 10^{-9}$  and  $5 \times 10^{-8}$  is predicted to be 0.39 and 0.64% respectively. These results show that high resolution thermometry is required to obtain an experimental accuracy of 1% in the critical exponent  $\delta$ .

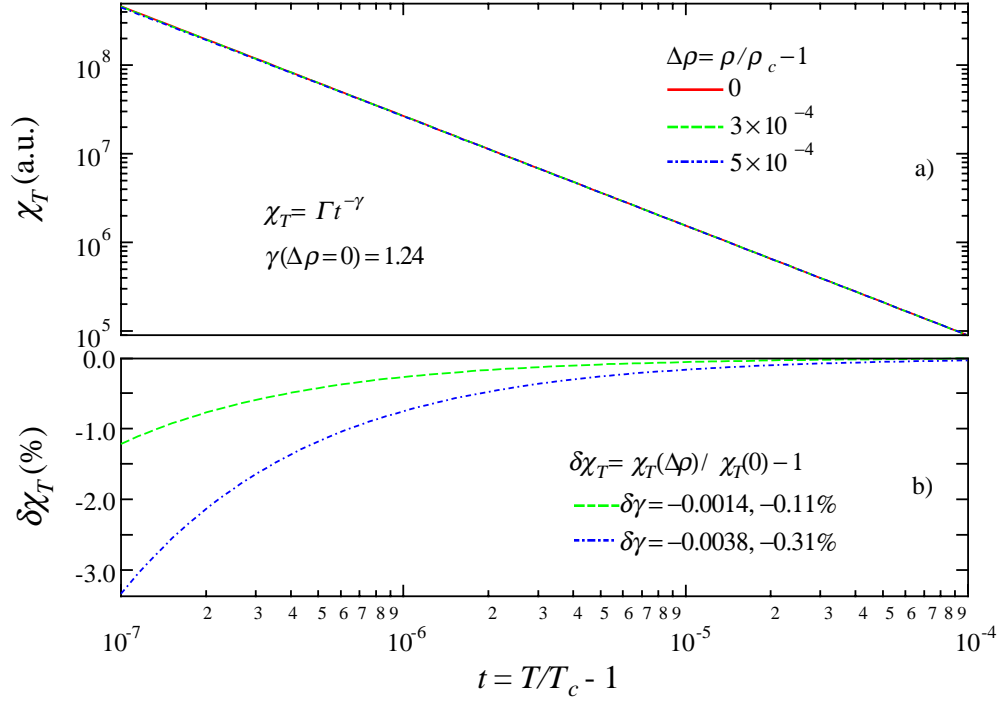


Fig. 2. Calculation of effective critical exponent  $\gamma$  for the linear susceptibility,  $\chi_T$ , along near critical isochore paths.

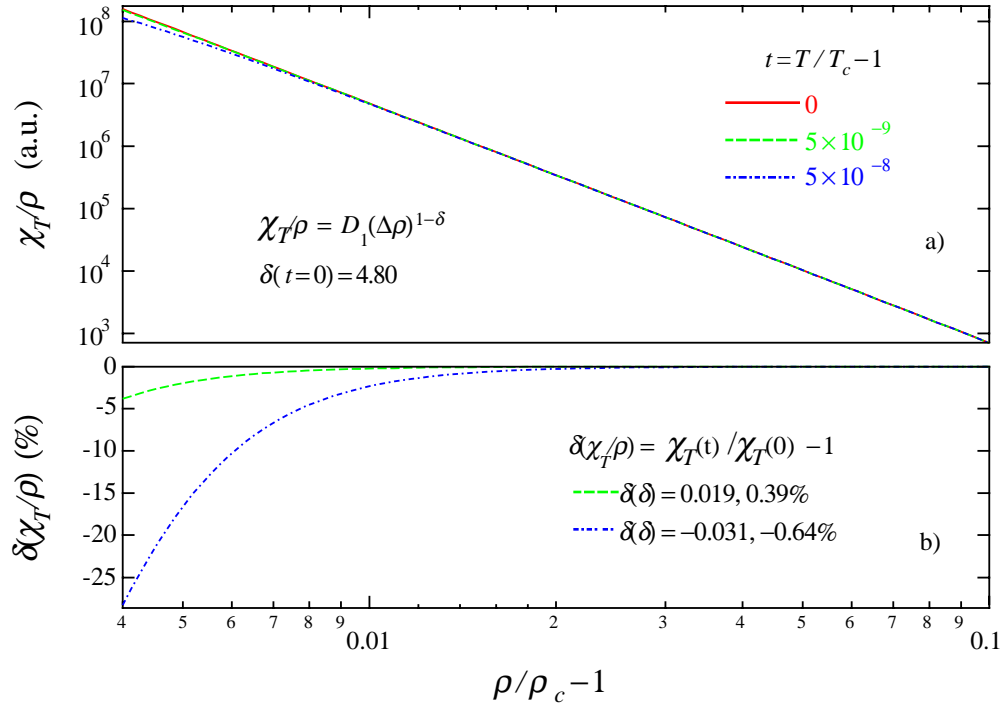


Fig. 3. Calculation of effective critical exponent  $\delta$  for  $\chi_T$  along near critical isotherm paths.

Fig. 4. Schematic showing heat transfer mechanisms between sample cell and shield stage.

The potential advantage of the drift method is the ability to perform  $C_V$  measurements upon cooling slowly through the critical point [4]. Figure 4 shows a schematic of the MISTE specific heat cell and surrounding shield stage. The main heat transfer mechanisms are conduction and radiation. The conduction path is primarily through the stainless steel supports between the cell and shield stage. Assuming a temperature difference  $\Delta T = (T_0 - T_1)$  between the cell and shield, the conduction thermal resistance,  $R_C$ , is given by

$$R_C = (T_0 - T_1) / (dQ_C / dt) , \quad (1)$$

where  $Q_C$  is the heat conducted away from the cell. For small temperature differences ( $\Delta T/T_C \ll 1$ ), one can similarly define a radiation thermal resistance  $R_r$  by

$$R_r = (T_0 - T_1) / (dQ_r / dt) , \quad (2)$$

with  $Q_r$  being the radiation energy leaving the cell. Using these thermal resistance definitions, the total heat capacity of the cell can be determined from the expression

$$C_T = -[(T_0 - T_1) / (dT_0 / dt)] [1 / R_C + 1 / R_r] . \quad (3)$$

It is assumed that the thermal resistances can be considered constant throughout the critical region. Thus, the precision in the measured total heat capacity is primarily dependent on the precision in measuring the cell and shield temperatures. At this time, the specific heat cell temperature is measured using a conventionally designed  $GdCl_3$  high resolution thermometer (HRT) while a prototype miniature  $GdCl_3$  HRT measures the shield temperature [5].

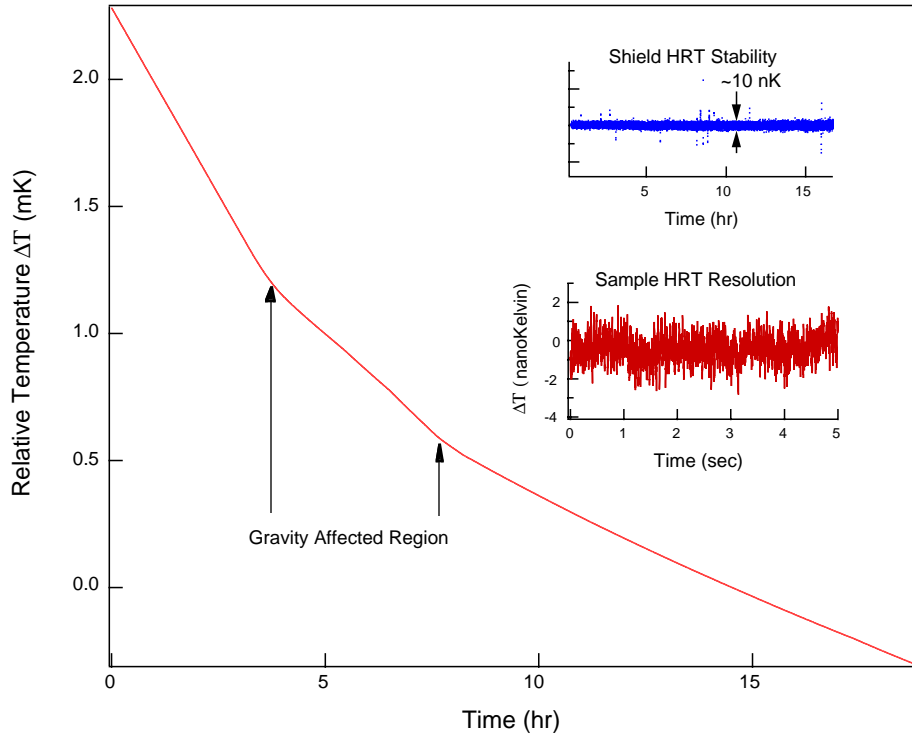


Fig. 5. Typical cooling drift experiment.

A typical cooling drift experiment slightly off the critical isochore is shown in Fig. 5. For this run, the shield temperature was held constant at  $\approx 2$  mK below the transition and the cooling rate was  $\approx 150$   $\mu\text{K/hr}$ . The changes in slope, indicated by the arrows, define the gravity affected region ( $t \approx 6 \times 10^{-4}$ ) near the phase transition. In a microgravity environment, this gravity induced smearing of the transition will be significantly reduced allowing more accurate measurements very near the transition. The inserts in the figure show the degree of stability and temperature resolution attained at the shield and cell stages. The shield temperature data shown here, which had no filtering, demonstrate that the shield stage could be held constant to much better than 10 nK for extended periods of time. The cell temperature data, with 100Hz filtering, show short term control to better than 1 nK. While further effort is required to develop the drift technique for measurements near the  $^3\text{He}$  critical point, the inserts in Fig. 5 demonstrate that the temperature resolution required for the MISTE precision specific heat measurements has been attained.

Preliminary specific heat measurements have also been performed using the pulse technique. Figure 6 shows a sample heat pulse measurement at a reduced temperature of  $3.5 \times 10^{-4}$ . The sample cell was drifting upward in temperature at a rate of  $\approx 3$  nK/sec. The temperature rise was determined by fitting the temperature versus time data above and below the pulse to a straight line. The uncertainty in the temperature difference  $\Delta T$ , between the drift rates above and below the pulse was  $\approx 0.3\%$ . This temperature resolution is sufficient for accurate pulse measurements to  $t \approx 10^{-7}$ .

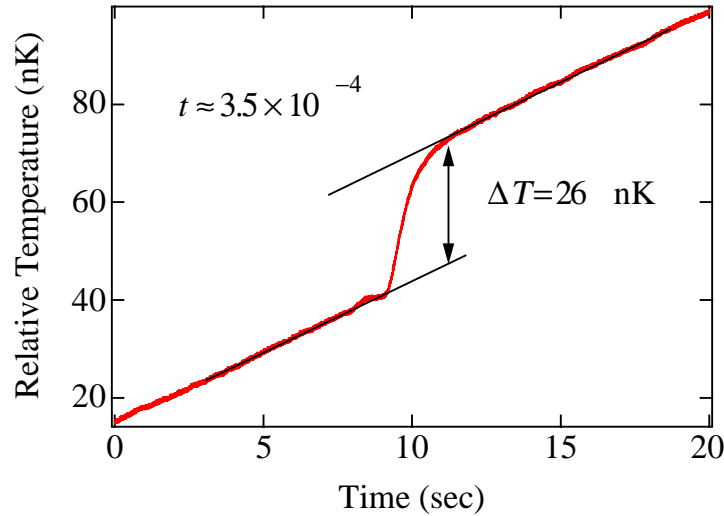


Fig. 6. Typical heat pulse measurement in the critical region.

### Conclusions

We have performed the first extensive low temperature evaluation of the ground-based MISTE specific heat sample cell. Preliminary measurements of the drift and pulse specific heat techniques have demonstrated that the required temperature resolution for a microgravity experiment were obtained. The miniature  $\text{GdCl}_3$  HRT provided the temperature resolution required for the shield stage. In the future, a similar miniature HRT will be tested on the sample cell. Replacing the conventional HRT design with a miniature unit will significantly reduce the volume and HRT calibration requirements of a future microgravity experiment. Future low temperature studies in the MISTE cell will evaluate electrostrictive techniques for measuring the isothermal compressibility in bulk [1] and finite size environments [6-9].

Theoretical estimates of the specific heat at constant volume and the linear susceptibility measured along near critical paths were also calculated. The maximum deviation from a critical isochore or critical isotherm path that will still permit 1% precision in the measured critical exponents  $\alpha$ ,  $\gamma$  and  $\delta$  was determined by fitting the theoretically generated “data” to the expected critical asymptotic power law expression.

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